

Higgs production in e^-e^+ collisions as a probe of noncommutativity

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Abstract

We examine the sensitivity of the angular distribution of the Higgs boson in the process of $e^+e^- \rightarrow ZH$ and the total cross section in the minimal noncommutative standard model (mNCSM) framework to set lower limit on the noncommutative characteristic scale (Λ). Contrary to the standard model case, in this process the Higgs boson tends to be emitted anisotropically in the transverse plane. Based on this fact, the profile likelihood ratio is used to set lower limit on Λ . The lower limit is presented as a function of the integrated luminosity. We show that at the center-of-mass energy of 1.5 TeV and with 500 fb^{-1} of data, the noncommutative characteristic energy scale Λ can be excluded up to 1.2 TeV.

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1 Introduction

The discovery of a Higgs-like boson by the ATLAS and CMS experiments [1] is considered as a milestone in illuminating the electroweak symmetry (EW) breaking mechanism. In case of observing no direct evidence for new physics at the LHC, one important task would be the precise measurement of the Higgs boson couplings with the Standard Model (SM) particles. In other words, the precise measurement in Higgs sector will be one of the main objectives of the future experiments if no new particle is found beside the Higgs boson. Beside the LHC, it has been shown

with detailed realistic simulations that the International Linear Collider (ILC) and the Compact Linear Collider (CLIC) can achieve high precision measurements for the Higgs boson properties [2, 3]. It is worth mentioning that the ILC [4] and CLIC [5] programs will be to run at the center-of-mass energies between 200 and 500 GeV to provide the opportunity for threshold scans like ZH , $t\bar{t}$, ZHH and $t\bar{t}H$. The ultimate goal of the ILC will be increasing the center-of-mass energy to 1 TeV while CLIC aims to reach at the center-of-mass energy of 3 TeV. Certainly, the LHC will improve the Higgs boson related measurements with more data that will be accumulated at the center-of-mass energies of 13 and 14 TeV. However, it is well known that the precise measurements at the ILC or CLIC are complementary to the LHC in many aspects [2, 3, 6].

As the nature of the space-time may change at Planck scale, a possible generalization of the ordinary quantum mechanics and quantum field theory to describe the physics at Planck scale is noncommutativity in space-time. Motivations for construction the models on noncommutative space-time are originating from the string theory, quantum gravity, and Lorentz breaking [7, 8, 9].

In the simplest way, the noncommutativity can be described by a set of constant c-number parameters $\theta_{\mu\nu}$ or equivalently can be characterized by an energy scale Λ and dimensionless parameters $C^{\mu\nu}$ as the following:

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = \frac{i}{\Lambda^2} C_{\mu\nu} \quad (1)$$

where $\theta^{\mu\nu}$ is an antisymmetric tensor with the dimension of $[M]^{-2}$.

A noncommutative version of the ordinary quantum field theory is obtained only by replacing the ordinary products with the so-called Moyal \star product that is defined as [10, 11]:

$$\begin{aligned} (f \star g)(x) &= \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^y\partial_\nu^z\right) f(y)g(z)\Big|_{y=z=x} \\ &= f(x)g(x) + \frac{i}{2}\theta^{\mu\nu}(\partial_\mu f(x))(\partial_\nu g(x)) + O(\theta^2). \end{aligned} \quad (2)$$

As mentioned, one can construct the noncommutative quantum field theory via Weyl correspondence in which the ordinary product among the fields is replaced by the Moyal \star product [11]. To study the noncommutative effects, we concentrate on the minimal version of the noncommutative SM [12]. By the means of Seiberg-Witten maps, one can expand the matter gauge fields in noncommutative space-time in terms of the commutative fields as power series of the noncommutativity parameter θ [11]. In the approach of Seiberg-Witten maps, the gauge fields A_μ and matter fields ψ in the noncommutative space-time can be expanded in terms of the commutative fields as power series of θ :

$$\hat{\psi}(x, \theta) = \psi(x) + \theta\psi^{(1)} + \dots \quad (3)$$

$$\hat{A}_\mu(x, \theta) = A_\mu(x) + \theta A_\mu^{(1)} + \dots \quad (4)$$

In the limit of $\theta \rightarrow 0$ the noncommutative fields reduce to the fields in the commutative space-time. One of the interesting advantage of this approach is that it can be applied to any gauge theory with arbitrary representation of matter field.

The minimal noncommutative SM predicts new interactions among the SM particles as well as correcting the ordinary SM vertices. This leads to interesting signals at the collider experiments. There are already several phenomenological studies on the effects of noncommutativity on various decay and scattering processes that can be found in [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24,

25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49]. In most of these studies, lower limits on the noncommutative characteristic scale have been set.

One interesting effects of noncommutativity is to change the angular distributions of the final state particles in the scattering processes and in the decay of unstable particles. It is because of the violation of the angular momentum conservation in the noncommutative theory. As an example, in [43] it has been shown that the noncommutativity affects the total cross section and the differential cross sections significantly in the $e^-e^+ \rightarrow ZH(HH)$ processes. Therefore, both the total and differential cross sections can be used to set lower limit on the noncommutative scale Λ . The same effect is present in $Z\gamma$ production that has been studied in [13] at the LHC and Tevatron.

As mentioned previously in [43] based on theoretical calculations, it has been found that the azimuthal distribution of the emitted Higgs boson in $e^-e^+ \rightarrow ZH$ process is sensitive to the noncommutativity. Now, the important task is to perform a more realistic study to obtain the possible limits on the noncommutative scale using this angular distribution and the total cross section at different center-of-mass energies and more importantly at different integrated luminosities of data that will be collected by the future e^-e^+ experiments. This should be done by employing advanced statistical methods that are used currently by the large experiments at the LHC and Tevatron. This will help us to know what would be the outcome of the future experiments at different phases of energy and luminosity.

The goal of this short report is to estimate the lower bound on the noncommutative scale Λ at 95% CL in electron-positron collisions with a Z-boson plus a Higgs boson in the final state. We set the limit using a test statistics based on the profile likelihood ratio [50] on the angular distribution. We also set limit on Λ by obtaining the upper limit on the total cross section of the signal using a Bayesian approach [51].

This article is organized as follows. In Section 2, the noncommutative cross section of the process $e^- + e^+ \rightarrow H + Z$ is shown. Section 3 is dedicated to use the profile likelihood ratio to extract the 95% CL lower limit on the noncommutative scale. In section 3, we also obtain the expected upper limit on the signal cross section including 1σ and 2σ bands. We show the limit as a function of the integrated luminosity.

2 Noncommutative cross section for production of a Higgs boson in association with a Z-boson

In this section we show the dependency of the total cross section of $e^+e^- \rightarrow HZ$ as a function of noncommutative scale (Λ) as well as the differential cross section $\frac{d\sigma}{d\phi}$ for different values of Λ . This differential distribution has been proposed in [43] to identify the noncommutative effects. The Feynman diagram for the $e^+e^- \rightarrow HZ$ process is shown in Fig.1. As it can be seen the process proceeds through s -channel via the exchange of a Z-boson. It is notable that the ZH final state can be produced via the exchange of Higgs boson that is not considered because of negligible electron Yukawa coupling.

The Feynman rule for the vertex ZZH is found to be [43]:

$$V_{\mu\nu,ZZH}(p,k,q) = \frac{im_Z^2}{v} \left\{ 2 \cos\left(\frac{1}{2}p\theta q\right) g_{\mu\nu} + \frac{1}{4}((\theta q)_\mu p_\nu + (\theta q)_\nu k_\mu) \times \left(\frac{\cos(\frac{1}{2}p\theta q) - 1}{p\theta q}\right) \right\} \quad (5)$$

where m_Z is the Z-boson mass, v is the vacuum expectation value. In the limit of $\theta \rightarrow 0$ the vertex ZZH goes to $\frac{2im_Z^2}{v}g_{\mu\nu}$ that is compatible with the SM. The corresponding matrix element has the following form:

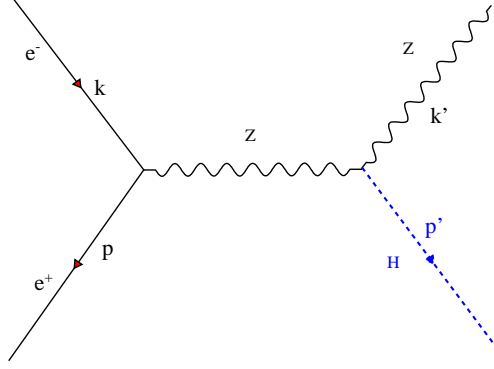


Figure 1: Feynman diagram for production of a Higgs boson in association with a Z-boson in electron-positron collisions.

$$\mathcal{M} \propto \bar{v}(p)\gamma_\mu(c_v - c_a\gamma_5)u(k)\frac{i}{s - m_Z^2 + i\Gamma_Z}V_{ZZH}^{\mu\nu}(k',p')\epsilon_\nu^*(k')e^{\frac{i}{2}p\theta k}, \quad (6)$$

where $c_v = -\frac{1}{2} + 2\sin\theta_W^2$, $c_a = -\frac{1}{2}$, and θ_W denotes the Weinberg angle. As it has been shown in Fig.1 k , p , k' and p' are the four-momenta of electron, positron, Higgs boson and Z boson, respectively. The center-of-mass energy is denoted by $\sqrt{s} = \sqrt{(k+p)^2} = \sqrt{(k'+p')^2}$ and Γ_Z is the width of the Z boson. To calculate the total and differential cross section, the equations of motion of the ingoing and outgoing particles are used. The mass of ingoing particles, m_e , is ignored in the calculations.

Then the cross section is calculated using the center-of-mass frame for $e^-(p) + e^+(k) \rightarrow H(p') + Z(k')$ process:

$$p^\mu = \frac{\sqrt{s}}{2}(1, 0, 0, 1) \quad , \quad k^\mu = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \quad (7)$$

$$p'^\mu = \frac{\sqrt{s}}{2}(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \quad , \quad k'^\mu = \frac{\sqrt{s}}{2}(1, -\sin\theta\cos\phi, -\sin\theta\sin\phi, -\cos\theta)$$

where θ is the polar angle and ϕ denotes the azimuthal angle. After some algebraic manipulations the total and differential cross sections are obtained. In all calculations in this work the mass of the Higgs boson is set to $m_H = 125$ GeV. The right plot of Fig.2 shows the relative correction from noncommutativity to the total cross section of $e^+e^- \rightarrow ZH$ at the center-of-mass energies of 1 and 1.5 TeV as a function of the noncommutative scale Λ . As it can be seen, the noncommutative correction increases with increasing the center-of-mass energy of the collisions. Because of the significant sensitivity of the total cross section, it can be used to set lower limits on the noncommutative scale.

In addition to the total cross section, on the left side of Fig.2 the differential cross section $d\sigma/d\phi$ at $\sqrt{s} = 1.5$ TeV is shown. From this plot, one can see that contrary to the SM case the distribution of $d\sigma/d\phi$ behaves like $\sin(\phi + \alpha)$. As it can be seen the noncommutativity leads the Higgs boson to be emitted in an anisotropic way in the transverse plane. This is due to the violation of the angular momentum conservation in our noncommutative model. An interesting observation is that with increasing the noncommutative characteristic scale the amplitude of the oscillation decreases and goes to zero while the minimum and maximum positions do not change. In the next section, we will set limit on the noncommutative scale using the total cross section and the oscillatory behaviour of the emitted Higgs bosons in the transverse plane.

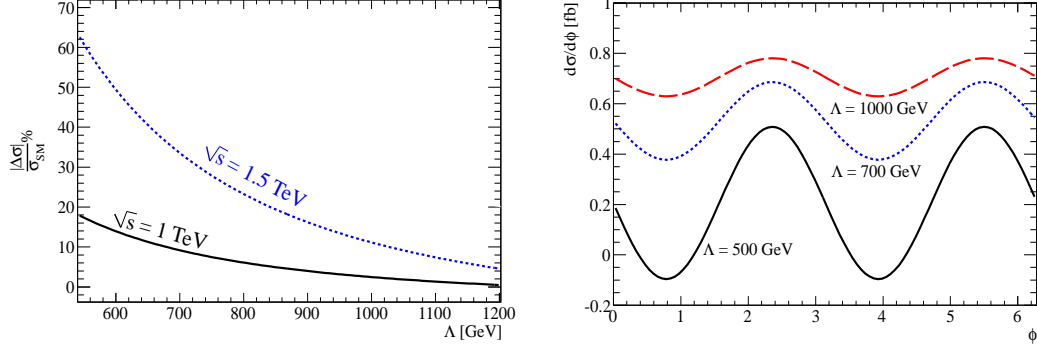


Figure 2: The relative correction from noncommutativity to the cross section of $e^+e^- \rightarrow ZH$ versus the noncommutative scale Λ (left). The differential cross section $\frac{d\sigma}{d\phi}$ for $e^+e^- \rightarrow ZH$ process at various values of Λ at $\sqrt{s} = 1.5$ TeV (right).

3 Test statistics and sensitivity estimates

Using the fact that the background, $e^-e^+ \rightarrow ZH$ within the SM ($\Lambda \rightarrow \infty$), has a quite different shape in the $d\sigma/d\phi$ distribution from the signal (a flat shape versus an oscillating behaviour), a test statistic can be constructed. Test statistic is a powerful tool to separate between signal (noncommutativity) and background and can enhance the separation power in comparison with other methods. More details can be found in [52]. We use the normalized ϕ distribution of the Higgs boson to define our test statistic as the profile likelihood ratio between the two hypotheses of *signal+background* and only *background*:

$$t = -2\ln(\lambda) \text{ with } \lambda = \frac{L(\mu = 1)}{L(\mu = 0)} \quad (8)$$

where L is defined as:

$$L = \prod_{bin\ i} P(n_i; \mu s_i + b_i) \quad (9)$$

where P is the Poisson distribution and s_i and b_i are the predicted numbers of signal and background events in bin i of the azimuthal distribution of the Higgs boson, respectively. We have histogrammed $d\sigma/d\phi$ distribution of the Higgs boson for the signal and background. In each bin of ϕ distribution $s_i = s_i(\Lambda) = \mathcal{L} \times d\sigma/d\phi_i$, in which \mathcal{L} denotes the integrated luminosity. The quantity λ is the profile likelihood ratio which means that for each one of the two values of $\mu = 1$ and $\mu = 0$, a fit is performed over the model parameter to find the value which maximizes the likelihood. The tools for construction of the test statistics have been implemented in the RooStats framework [53] that is a C++ class library based on the RooFit [55] and ROOT [54] programs. The output is the limit on the model parameter that is the noncommutative characteristic scale Λ . The results are shown in Fig.3. It shows the lower limit on the noncommutative scale as a function of the integrated luminosity. As it can be seen the lower limit on Λ grows up to around 1.2 TeV when the integrated luminosity reaches around 500 fb⁻¹. Then with increasing the integrated luminosity no improvement on the lower limit on Λ is observed. This is because of the fact that for $\Lambda \sim 1.2$ TeV and larger values, the oscillating behaviour in ϕ distribution looks like the SM background distribution considering the uncertainties. It should be mentioned here that this estimation is idealistic as no detector simulation has been performed. In addition, other backgrounds

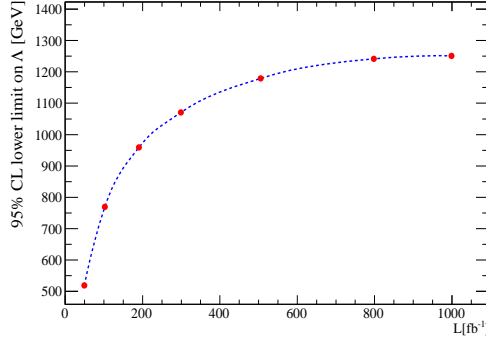


Figure 3: The 95% C.L. lower limits on Λ as a function of the integrated luminosity in electron-positron collider with $\sqrt{s} = 1.5$ TeV using the azimuthal angular distribution of the Higgs boson without including any systematic effects.

Integrated luminosity	100 fb ⁻¹	500 fb ⁻¹	1000 fb ⁻¹
limit on Λ : shape analysis ($d\sigma/d\phi$)	0.67 TeV	1.05 TeV	1.11 TeV
limit on Λ : total cross section	0.62 TeV	0.94 TeV	1.10 TeV

Table 1: The lower limit on the noncommutative scale at two integrated luminosities of 100, 500 and 1000 fb⁻¹ using the shape of ϕ distribution of the Higgs boson and the total cross section.

like ZZ, W^+W^- have not been considered as well as all theoretical and instrumental systematic uncertainties. After considering all the effects one would expect the limits to be looser. To have a rough estimation of the detector and systematic effects we vary the number of events in each bin of ϕ distribution by $\pm 10\%$ to consider these effects as well as the background shape uncertainty and then recalculate the limit. We apply a Gaussian smearing on each bin of the standard model ϕ distribution in order to consider an overall systematic uncertainties which change the shape of ϕ distribution. Using $G(m, \sigma)$ a random number that belongs to a Gaussian distribution with a mean value of m and a standard deviation σ , the number of events in each bin of ϕ distribution will be smeared as:

$$N_{smeared}(\phi) = \mathcal{L} \times \frac{d\sigma}{d\phi} \times G(1, \Delta) \quad (10)$$

where \mathcal{L} is the integrated luminosity and Δ is set to 10% as discussed previously. We found that the lower limit on Λ decreased to around 1.05 TeV using 500 fb⁻¹ of data. To obtain a realistic estimation of the sensitivity, the backgrounds, detector effects and selection cuts have to be fully considered. The analysis of all backgrounds and simulation of detector effects is beyond the scope of this short report and must be done by the experimental collaborations.

Another way to set limit on the model parameter is use the total cross section of the signal. By assuming conservative values for the number of backgrounds and the efficiencies we can set upper limit on the signal cross section ($\sigma_{NC}(e^-e^+ \rightarrow ZH)$). Then the upper limit on the signal cross section can be translated on the lower bound on the noncommutative scale (Λ). In the absence of a significant excess above the expected background at any given integrated luminosity of data, one can proceed with setting limits on the model parameter Λ . To calculate the upper limits on the signal cross section, a counting experiment is performed. We exploit a standard Bayesian approach [51] with a flat prior that is chosen for the signal cross section. More details of the

\sqrt{s}	0.5 TeV	1 TeV	1.5 TeV
limit on Λ : shape analysis ($d\sigma/d\phi$)	0.34 TeV	0.69 TeV	1.05 TeV
limit on Λ : total cross section	0.31 TeV	0.63 TeV	0.94 TeV

Table 2: The lower limit on the noncommutative scale with the integrated luminosity of 500 fb^{-1} using the shape of ϕ distribution of the Higgs boson and the total cross section for the center-of-mass energies of 0.5,1.0,1.5 TeV.

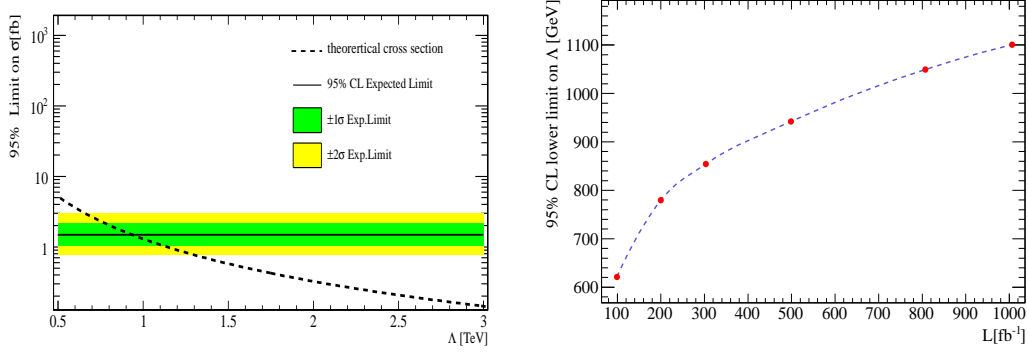


Figure 4: The expected 95% CL upper limit on the signal cross section using 500 fb^{-1} (left). The expected limit is compared with the theoretical prediction. The uncertainty band on the limits at 1σ and 2σ are shown with shaded bands around the limit. The 95% CL lower limits on Λ as a function of the integrated luminosity in electron-positron collider with $\sqrt{s} = 1.5$ TeV (right).

statistical method can be found in [50]. All the calculations are performed with the RooStats [53] calculator for the expected limit.

In limit setting process, we choose the conservative numbers of backgrounds and efficiencies based on the latest LEP analysis in search for the Higgs boson in ZH channel [56]. We assume the number of survived background events to be twice of the signal ($n_s/n_b = 0.5$) and an efficiency of signal to be 85% with an uncertainty of 5%. For simplicity, the efficiency is assumed to be fixed for different values of Λ . In the left side of Fig.4 the expected 95% CL upper limit on the signal cross section is shown. The expected limit is compared with the theoretical prediction. The uncertainty bands on the limit at 1σ and 2σ are shown with shaded bands around the limit. The 95% CL lower limits on Λ as a function of the integrated luminosity are shown in right side of Fig.4. Clearly, with increasing the amount of data the lower limit on the noncommutative scale is increased. Table 1 compares the lower bound on Λ obtained from the shape analysis and the upper limit on the signal cross section for 100, 500 and 1000 fb^{-1} of data at $\sqrt{s} = 1.5$ TeV.

In table 2, we show the lower limit on the noncommutative scale with the integrated luminosity of 500 fb^{-1} using the shape of ϕ distribution of the Higgs boson and the total cross section for the center-of-mass energies of 0.5,1.0,1.5 TeV. As it can be seen, the electron-positron collisions with higher center-of-mass energy provide stronger limit on the noncommutative characteristic scale Λ .

An interesting feature of noncommutativity that could be considered is to study the effect of earth rotation on the cross section of a process. It leads the cross section to be a time-dependent observable since the detector orientation changes with the earth rotation. However, since it is troublesome to have access to the time dependent data for the cross section measurement, the time-averaged cross section is considered to examine the earth rotation effect. This average

is over the sidereal day which 23 hours, 56 minutes and 4.091 seconds. For example, in [57] the authors have studied the effect of rotation of earth on the cross section of $e^-e^+ \rightarrow HH$ process. It has been shown that the time-averaged cross section can deviate from the SM prediction around 15%. The earth rotation effect on the $e^-e^+ \rightarrow HZ$ process also could be at similar order while precise calculation is needed. It is interesting to point out here that in [58], the DØ collaboration performed a search for the Lorentz violation based on the standard model extension framework (SME) [59]. Similar to noncommutative SM, it predicts that the cross sections are dependent on sidereal time as the detector orientation changes with the earth rotation. The DØ collaboration performed the search on the $t\bar{t}$ events based on the SME. Within the uncertainties no time dependent effect on the cross section has been observed.

4 Conclusions

In this letter we have concentrated on Higgs plus Z-boson production at a future electron-positron collider to explore the sensitivity of future accelerator experiments to the noncommutativity. The noncommutativity destructs the isotropic azimuthal angular distribution of final state particles. We used this feature to search for the signal of noncommutative theory using a test statistic technique. Furthermore, by using a Bayesian approach with some conservative assumptions for the backgrounds and efficiencies, conservative estimates obtained on the noncommutative scale. We find that the Higgs boson angular distribution shape shows more sensitivity to the model parameter than the total cross section. It is shown that in this channel, the lower limit of 1.1 TeV on Λ can be achieved using 500 fb^{-1} of data in electron-positron collisions at the center-of-mass energy of 1.5 TeV.

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